Devising New Australian Rules Football Scoring Systems
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Abstract
In every AFL season, there are usually one or two round matches that end in a draw. This situation is handled in finals matches by playing extra time. Whilst the scoring systems used in finals matches could be applied to round matches to eliminate draws, we devise new alternative systems that have better statistical properties by increasing the probability for the stronger team to win, increasing fairness and spectator interest, and reducing the likelihood of long matches occurring. Further, these new scoring systems could also be used in finals matches.

KEYWORDS: Markov Chain model, Australian Rules football, AFL, probability

1. Introduction
In sporting events involving a fixed time duration of play, there is always the possibility of the result ending in a draw. Where a win must be obtained by a team (such as a final, teams advancing to the next round or in qualifying matches), different methods of breaking the tie are adopted. In football, if scores are level after extra time, the well known penalty shoot-out is used to obtain a winner. Often extra time is given in sporting events to break the tie, and extra time may be given indefinitely until a winner is obtained. In finals matches (excluding the grand final) of Australian Rules football (AFL), an extra 10 minutes is given if teams are level after the fixed 80 minutes of playing time. If teams are level after the extra 10 minutes, then this process repeats until a winner is identified. In the grand final match of AFL no extra time is given, and the entire match is played again the following week if teams are level after the fixed 80 minutes of play. If teams are level after the replayed match, then extra time will be played as given above for finals matches. In regular season round matches of AFL, no extra time is given if teams are level after the fixed 80 minutes of play, and the match is declared a draw with both teams being awarded 2 points each (a win in a round AFL match results in 4 points to the winner only). Note that AFL matches average 116 minutes of total time since the clock is stopped when the ball is not in play i.e. going out of the boundary of play. The model and analysis to follow will use a fixed 80 minutes of play and not consider stoppages.

Should extra time be given in AFL round matches to guarantee that there are no draws? This issue is subjective but there may be enough support from
coaches and spectators to warrant extra time. For example, in round 16 of the 2009 series, Richmond and North Melbourne drew level at 85 points each after 80 minutes of play. Both the Richmond and North Melbourne coaches felt that extra time should be given to split teams that are level on the siren. It would also appear from a spectator’s perspective that extra time would create additional entertainment.


In this paper we study the characteristics of the probabilities of each team winning and the distribution of the number of minutes in the match, if extra time was adopted in AFL round matches to stop draws occurring. This is achieved by comparing the characteristics of the current scoring systems used in AFL matches with our alternative scoring systems. Further, these new scoring systems could also be used as alternative systems in finals matches.

2. AFL Scoring System in Round Matches

Two independent competitions currently exist in elite AFL, consisting of the pre-season competition (known as the NAB Cup) and the Premiership Season. The analysis in the paper applies only to the Premiership Season, as different amount of points are awarded for scoring shots in both competitions.

There are three scoring systems currently used in Premiership Season AFL matches, with different systems in the rounds, the finals (excluding the grand final) and the grand final. For notational purposes we will adopt other finals matches to refer to all finals matches excluding the grand final. In all systems, a team scores 6 points for a goal and 1 point for a behind. Round matches are analyzed in this section with finals scoring systems to follow in section 3.

Several authors have investigated models for predicting outcomes in AFL and other football codes. Croucher (2002) analyzed scores in English premier league soccer and found that the negative binomial distribution yields a better fit of goal scoring than does the Poisson distribution. Glasson (2006) uses a Brownian motion model to estimate a team’s chance of winning an AFL match that is already underway. We choose to develop a Markov Chain model to obtain numerical results, by allocating one scoring shot in a short specified time period.

2.1 Markov Chain Model

The states of the model are defined by the margin (Team A is d points ahead of Team B) and time (T minutes remaining in the match).
The parameters of the model are:

Let \( p_{A6} \) and \( p_{B6} \) be the probabilities of Team A and Team B scoring a goal in a 1-minute time period respectively.

Let \( p_{A1} \) and \( p_{B1} \) be the probabilities of Team A and Team B scoring a behind in a 1-minute time period respectively.

Let \( p_{ns} \) be the probability of no score in a 1-minute time period.

Let us now define our assumptions:

1) \( p_{A6} + p_{B6} + p_{A1} + p_{B1} + p_{ns} = 1 \)

2) Independence holds for each time interval such that only one scoring shot (by either Team A or Team B), or no scoring shot can occur in a 1-minute time period.

3) Time is bounded by \( 0 \leq T \leq 80 \)

4) Margin is bounded by \(-480 \leq d \leq 480\), since a team scoring 6 points (one goal) every minute for 80 minutes is 480 points.

The distribution of margins, the total number of points for team A, the total number of points for team B and the total number of points for both teams combined are now obtained for round matches using recursive formulas. The probabilities of winning for Team A and Team B can be obtained from the distribution of margins.

2.2 Distribution of Margins

Let \( P_1(d,T) \) be the probability of Team A reaching a margin of \( d \) points ahead of Team B with \( T \) minutes remaining in the match.

The boundary condition is given by:

\[ P_1(0,80) = 1. \]

The recurrence formula becomes

\[ P_1(d,T) = p_{A6}P_1(d-6,T+1) + p_{B6}P_1(d+6,T+1) + p_{A1}P_1(d-1,T+1) + p_{B1}P_1(d+1,T+1) + p_{ns}P_1(d,T+1), \text{ if } -480 \leq d \leq 480 \text{ and } 0 \leq T \leq 80 \]

The distribution of margins for the match is obtained from \( P_1(d,0) \), for \(-480 \leq d \leq 480\).

Let \( w_{AR} \) and \( w_{BR} \) be the probabilities of Team A and Team B winning a round match (between Team A and Team B), and \( w_{DR} \) be the probability of a drawn match.
w_{AR} = \sum_{0<d\leq480} P_1(d,0), \quad w_{BR} = \sum_{480<d<0} P_1(d,0) \quad \text{and} \quad w_{DR} = P_1(0,0).

It follows that $w_{AR} + w_{BR} + w_{DR} = 1$.

2.3 Distribution of the Total Number of Points for a Particular Team

Let $P_{2A}(d,T)$ and $P_{2B}(d,T)$ be the respective probabilities of Team A and Team B reaching a point score of $d$ points with $T$ minutes remaining in the match. The boundary conditions are given by:

$P_{2A}(0,80) = P_{2B}(0,80) = 1.$

The recurrence formulas become

$P_{2A}(d,T) = (1 - p_{A1} - p_{A6}) P_{2A}(d,T+1)$, if $d=0$ and $0 \leq T \leq 79$

$P_{2A}(d,T) = (1 - p_{A1} - p_{A6}) P_{2A}(d,T+1) + p_{A1} P_{2A}(d-1,T+1)$, if $1 \leq d \leq 5$ and $0 \leq T \leq 79$

$P_{2A}(d,T) = (1 - p_{A1} - p_{A6}) P_{2A}(d,T+1) + p_{A1} P_{2A}(d-1,T+1) + p_{A6} P_{2A}(d-6,T+1)$, if $6 \leq d \leq 480$ and $0 \leq T \leq 79$

$P_{2B}(d,T) = (1 - p_{B1} - p_{B6}) P_{2B}(d,T+1)$, if $d=0$ and $0 \leq T \leq 79$

$P_{2B}(d,T) = (1 - p_{B1} - p_{B6}) P_{2B}(d,T+1) + p_{B1} P_{2B}(d-1,T+1)$, if $1 \leq d \leq 5$ and $0 \leq T \leq 79$

$P_{2B}(d,T) = (1 - p_{B1} - p_{B6}) P_{2B}(d,T+1) + p_{B1} P_{2B}(d-1,T+1) + p_{B6} P_{2B}(d-6,T+1)$, if $6 \leq d \leq 480$ and $0 \leq T \leq 79$

The distribution of the total number of points for Team A for the match is obtained from $P_{2A}(d,0)$, for $0 \leq d \leq 480$.

The distribution of the total number of points for Team B in the match is obtained from $P_{2B}(d,0)$, for $0 \leq d \leq 480$.

2.4 Distribution of the Total Number of Points for the Match

Let $P_3(d,T)$ be the probability of reaching a total point score (both teams combined) of $d$ points with $T$ minutes remaining in the match. The boundary condition is given by:

$P_3(0,80) = 1.$

The recurrence formulas become

$P_3(d,T) = p_{ns} P_3(d,T+1)$, if $d=0$ and $0 \leq T \leq 79$

$P_3(d,T) = p_{ns} P_3(d,T+1) + (p_{A1} + p_{A2}) P_3(d-1,T+1)$, if $1 \leq d \leq 5$ and $0 \leq T \leq 79$
\[ P_3(d, T) = p_{a1}P_3(d, T+1) + (p_{a1}+p_{a2})P_3(d-1, T+1) + (p_{a6}+p_{b6})P_3(d-6, T+1), \text{ if } 6 \leq d \leq 480 \text{ and } 0 \leq T \leq 79 \]

The distribution of the total number of points for the match (both teams combined score) is obtained from \( P_3(d, 0) \), for \( 0 \leq d \leq 480 \).

3. AFL Scoring Systems in Finals Matches

The Markov Chain model developed in section 2.1 applies to round matches. The differences in win probabilities and match duration from finals matches, arise from the extra time given at the completion of 80 minutes of play to eliminate the possibility of a draw occurring.

3.1 Other Finals

Let \( w_{A10} \) and \( w_{B10} \) represent the probabilities of Team A and Team B being ahead after 10 minutes of play and \( w_{D10} \) represent the probability that the scores are equal after 10 minutes of play.

\[
\begin{align*}
  w_{A10} & = \sum_{0<d\leq480}P_1(d,70), \\
  w_{B10} & = \sum_{480\leq-d<0}P_1(d,70) \quad \text{and} \\
  w_{D10} & = P_1(0,70)
\end{align*}
\]

Let \( w_{AF} \) and \( w_{BF} \) be the probabilities of Team A and Team B winning other finals matches (between them).

Then \( w_{AF} = w_{AR} + w_{DR}w_{A10} / (1 - w_{D10}) \), and

\[ w_{BF} = 1 - w_{AF} \]

Let \( P_4(T_1) \) represent the probability of other finals matches going for \( T_1 \) minutes in duration given teams are level after 80 minutes of play.

\[ \text{Then } P_4(T_1) = P_1(0,70)^{(T_1-10)/10}, \text{ for } T_1=10,20,30..... \]

3.2 Grand Final

Let \( w_{AG} \) and \( w_{BG} \) be the probabilities of Team A and Team B winning a grand final match (between them).

\[ \text{Then } w_{AG} = w_{AR} + w_{DR}w_{AF}, \text{ and } w_{BG} = 1 - w_{AG}. \]

4. Data

In order to make a comparison between systems, reasonable values for the parameters of the model need to be obtained. The official AFL website [www.afl.com.au](http://www.afl.com.au) provides past match results from 1897. We chose to analyze the match score lines from the 2008 Premiership season. A total of 185 matches were analyzed.
played comprising 176 matches from the 22 rounds and the 9 matches from the finals. Two of these matches ended in a draw. More specifically, the Western Bulldogs drew 19-16-130 against Richmond 20-10-130 in the 5th round and North Melbourne drew 9-10-64 against Sydney 8-16-64 in the 6th round. Note that in both of these matches one of the teams scored fewer goals than their opponent but the match nevertheless ended in the draw. Two of the 176 matches resulted in a win even though fewer goals were scored by the winner. More specifically, Hawthorn 14-12-106 defeated Richmond 15-4-96 in the 6th round and Adelaide 9-20-74 defeated Essendon 10-9-69 in the 10th round. In the 185 matches that were played, 5238 goals and 4609 behinds were scored. This was an average of 28.31 goals and 24.91 behinds per match, or 14.16 goals and 12.46 behinds/per team/per match. For the winning team (excluding the two drawn matches) an average of 17.02 goals and 13.43 behinds were scored per match and for the losing team an average of 11.30 goals and 11.48 behinds were scored per match.

5. Comparison of Current AFL Scoring Systems

In order to make comparisons of scoring systems it is necessary to evaluate various statistical characteristics. We will study the following relevant characteristics a) probability of the superior team winning, b) match duration, c) fairness. Spectator interest is also a consideration.

Table 1 gives the probabilities of teams winning and drawing matches for all three scoring systems. The parameters used are based on the averages from section 4 and are as follows:

(1) $p_{A6} = 0.177$, $p_{B6} = 0.177$, $p_{A1} = 0.156$, $p_{B1} = 0.156$, $p_{ns} = 0.334$, and
(2) $p_{A6} = 0.213$, $p_{B6} = 0.141$, $p_{A1} = 0.168$, $p_{B1} = 0.143$ $p_{ns} = 0.335$.

<table>
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<tr>
<th>Parameters</th>
<th>Rounds</th>
<th>Other Finals</th>
<th>Grand Final</th>
</tr>
</thead>
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<tr>
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<td>$w_{AR}$</td>
<td>$w_{BR}$</td>
<td>$w_{DR}$</td>
</tr>
<tr>
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<td>0.494</td>
<td>0.494</td>
<td>0.012</td>
</tr>
<tr>
<td>(2)</td>
<td>0.870</td>
<td>0.124</td>
<td>0.006</td>
</tr>
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</table>

Table 1: The probabilities of teams winning and drawing matches for all of the currently used AFL scoring systems for two sets of parameter values

The results from Table 1 show that if both teams are evenly matched, then on average 1 in every 81 matches will end in a draw after 80 minutes of play. However, if one team is superior to the other, then on average 1 in every 154
matches will end in a draw. Therefore, it can be expected that 1 to 2 round matches in a season will result in a draw.

In the grand final match, the entire match is played again the following week if teams are level after the fixed 80 minutes of play. If teams are level after the replayed match, then extra time will be played as given for other finals matches. Therefore the probability of a grand final match lasting for 160 minutes is obtained as \( w_{DR} \). This is given by 0.012 for teams equal in ability and 0.006 when one team is superior over the other. These values are significantly greater than the probability of playing for 160 minutes in others finals matches, as reflected from table 2. From table 1, the probability of the superior team winning other finals matches (0.874) is about the same as that of winning a grand final match (0.875), yet the overall match duration is less for other finals matches. It could also be argued that replaying matches on the following week (if a draw is obtained) may cause complaints from spectators, given that spectators have paid for an event that has not been completed. Therefore, it could be argued that the system used in other finals matches is better than the system used in grand final matches since, although it has a slightly smaller probability of correctly identifying the better team, the expected period of time is reduced, and is possibly also better for spectator interest.

Table 2 gives the probabilities of other finals matches going for \( T_1 \) minutes in duration given teams are level after 80 minutes of play. These values can be used as a guide to determine whether the scoring system used in other finals matches could be applied to round matches to eliminate any possibility of a draw.

<table>
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</tr>
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<td>0.00222</td>
<td>0.00010</td>
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<tr>
<td>(2)</td>
<td>0.04358</td>
<td>0.00190</td>
<td>0.00008</td>
</tr>
</tbody>
</table>

Table 2: The probabilities of other finals matches match going for \( T_1 \) minutes in duration given teams are level after 80 minutes of play

6. Alternative Scoring Systems

Markov Chain models have been widely used in modeling sporting outcomes. For example, Barnett and Clarke (2005) applied Markov Chain models to tennis in order to estimate probabilities of winning as well as match duration. Barnett et al. (2008) applied Markov Chain models to volleyball in order to estimate match outcomes. It was noted that volleyball has an extra complexity on tennis, as the rotation of serve depends on which team won the previous point.
Our alternative scoring systems for AFL involve the following: Team A starts at $n$ points ahead of Team B. A team scores one point for a behind and six points for a goal. A team must be ahead by $N > n$ points to win (for $N > n$). Note that a draw cannot occur with these types of scoring systems.

### 6.1 Markov Chain Model

Let $q_{A1}$ and $q_{A6}$ represent the probability of Team A scoring a behind and goal as the next scoring shot respectively, and $q_{B1}$ and $q_{B6}$ represent the probability of Team B scoring a behind and goal as the next scoring shot respectively.

Because the events in the chain occur at a scoring shot it is no longer necessary to keep track of time, as in section 2.1. However the model developed here is related to the model in that section, and we can use the same data to derive its parameters. It is easy to show that

\[
q_{A1} = p_{A1} / (1 - p_{ns}), \\
q_{A6} = p_{A6} / (1 - p_{ns}), \\
q_{B1} = p_{B1} / (1 - p_{ns}), \quad \text{and} \\
q_{B6} = p_{B6} / (1 - p_{ns}).
\]

Note that $q_{A1} + q_{A6} + q_{B1} + q_{B6} = 1$, and the assumption of the independence of scoring shots (from one to the next) still holds.

The formulation of this model is given by the backwards recurrence formula

\[
R(d, m) = q_{A6} R(d+6, m+1) + q_{B6} R(d-6, m+1) + q_{A1} R(d+1, m+1) + q_{B1} R(d-1, m+1),
\]

for $|d| < N$ and $m \geq 0$, where $R(d, m)$ is the probability of Team A winning with a margin of $d$ points ahead of Team B.

If we set the boundary conditions as

\[
R(d, m) = 1 \text{ for } N \leq d \leq N+5 \text{ and } 0 < m \leq M, \\
R(d, m) = 0 \text{ for } -N-5 \leq d \leq -N \text{ and } 0 < m \leq M, \text{ and} \\
R(d, M) = 0 \text{ for } |d| < N,
\]

then $w_{AN} \approx R(0, 0)$.

Likewise, if we set the boundary conditions as

\[
R(d, m) = 0 \text{ for } N \leq d \leq N+5 \text{ and } 0 < m \leq M, \\
R(d, m) = 1 \text{ for } -N-5 \leq d \leq -N \text{ and } 0 < m \leq M, \text{ and} \\
R(d, M) = 0 \text{ for } |d| < N
\]

then $w_{BN} \approx R(0, 0)$. 
These approximations can be improved by replacing the boundary values of \( R(d,M) \) by \( R(d,0) \) and iterating. This iterative method can be shown to work even for the case \( M=1 \).

Table 3 gives the probabilities of Team A winning for the alternative scoring systems for \( N = 1 \) to 6 and \( q_{A6} = 0.320, q_{A1} = 0.253, q_{B6} = 0.212, q_{B1} = 0.215 \). These values are the equivalence of parameters (2).

<table>
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<th>( N )</th>
<th>( n=5 )</th>
<th>( n=4 )</th>
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<th>( n=1 )</th>
<th>( n=0 )</th>
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<th>( n=-2 )</th>
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<td></td>
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</tr>
<tr>
<td>4</td>
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<td>0.57</td>
<td>0.53</td>
<td>0.52</td>
<td>0.50</td>
<td>0.46</td>
<td>0.30</td>
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<td>5</td>
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<td>0.49</td>
<td>0.41</td>
<td>0.34</td>
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<tr>
<td>6</td>
<td>0.79</td>
<td>0.72</td>
<td>0.69</td>
<td>0.66</td>
<td>0.61</td>
<td>0.52</td>
<td>0.43</td>
<td>0.38</td>
<td>0.35</td>
<td>0.32</td>
<td>0.24</td>
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Table 3: The probabilities of Team A winning for the alternative scoring systems with \( N = 1 \) to 6, and \( q_{A6}=0.320, q_{A1}=0.253, q_{B6}=0.212 \) and \( q_{B1}=0.215 \)

**7. New AFL Scoring Systems**

The following new AFL scoring systems are proposed:

Both teams start at 0 points each. A team scores 6 points by scoring a goal and a team scores 1 point by scoring a behind. A team must be ahead by at least \( i \) points after 80 minutes of play to win the match. Otherwise play continues indefinitely until one team is ahead by at least \( j \) points \( (i\leq j) \). If \( i=0 \), the system is equivalent to the current AFL scoring system in the rounds and a draw is possible. For notational purposes, these sets of systems are referred to by \( L_{ij} \).

**7.1 Scoring Systems \( L_{ij} \) (i=1)**

For the set of systems \( L_{ij} \) (i=1), play continues only if teams are level after 80 minutes of play.

Table 4 represents the probabilities of Team A winning the match for scoring systems \( L_{ij} \) (j=1 to 6) conditional on the teams being level after 80 minutes of play using Parameters (2). It shows that the stronger team (Team A) is favoured by the requirement of being 6 points ahead in order to win the match \( (L_{16} = 0.618) \) when compared to being one point ahead to win the match \( (L_{11} = 0.573) \).
Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$L_{11}$</th>
<th>$L_{12}$</th>
<th>$L_{13}$</th>
<th>$L_{14}$</th>
<th>$L_{15}$</th>
<th>$L_{16}$</th>
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<tbody>
<tr>
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<td>0.602</td>
<td>0.603</td>
<td>0.607</td>
<td>0.618</td>
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</table>

Table 4: Probabilities of Team A winning the match for new AFL scoring systems if teams are level after 80 minutes of play

Tables 5 and 6 represent the probabilities of the match going for more than $T_1$ minutes in duration given teams are level after 80 minutes of play using Parameters (1) and Parameters (2) respectively. Table 7 represents the average time played in minutes if teams are level after 80 minutes of play. A 2000 run simulation was used to obtain the results for tables 5, 6 and 7.

Table 5: Probabilities of the match going for more than $T_1$ minutes in duration given teams are level after 80 minutes of play and Parameters (1) are applied

<table>
<thead>
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<th>Minutes</th>
<th>$L_{11}$</th>
<th>$L_{12}$</th>
<th>$L_{13}$</th>
<th>$L_{14}$</th>
<th>$L_{15}$</th>
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</tr>
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<tr>
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<td>0.101</td>
<td>0.108</td>
<td>0.137</td>
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<tr>
<td>10</td>
<td>0.000</td>
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</tr>
<tr>
<td>15</td>
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<td>20</td>
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<td>0.000</td>
<td>0.001</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table 6: Probabilities of the match going for more than $T_1$ minutes in duration given teams are level after 80 minutes of play and Parameters (2) are applied

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$L_{11}$</th>
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<th>$L_{13}$</th>
<th>$L_{14}$</th>
<th>$L_{15}$</th>
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<td>2.7</td>
<td>2.8</td>
<td>3.1</td>
<td>3.7</td>
</tr>
<tr>
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<td>2.5</td>
<td>2.7</td>
<td>2.9</td>
<td>2.9</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Table 7: Average time played in minutes if teams are level after 80 minutes of play
7.2 Scoring Systems $L_{ij}$ (i=j)

Table 8 represents the probabilities of Team A winning the match for scoring systems $L_{ij}$ (i=j and i=1 to 6) and table 9 represents the probabilities of playing more time after the fixed 80 minutes of play.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$L_{11}$</th>
<th>$L_{22}$</th>
<th>$L_{33}$</th>
<th>$L_{44}$</th>
<th>$L_{55}$</th>
<th>$L_{66}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)</td>
<td>0.873</td>
<td>0.874</td>
<td>0.875</td>
<td>0.876</td>
<td>0.878</td>
<td>0.880</td>
</tr>
</tbody>
</table>

Table 8: Probabilities of Team A winning the match for new AFL scoring systems

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$L_{11}$</th>
<th>$L_{22}$</th>
<th>$L_{33}$</th>
<th>$L_{44}$</th>
<th>$L_{55}$</th>
<th>$L_{66}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.012</td>
<td>0.037</td>
<td>0.062</td>
<td>0.086</td>
<td>0.111</td>
<td>0.135</td>
</tr>
<tr>
<td>(2)</td>
<td>0.006</td>
<td>0.019</td>
<td>0.032</td>
<td>0.045</td>
<td>0.058</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Table 9: Probabilities of playing more time after the fixed 80 minutes of play

8. Discussion

An analysis of the current AFL scoring systems was given in section 5. The results from table 1 show that if both teams are evenly matched, then on average 1 in every 81 matches will end in a draw after 80 minutes of play. However, if one team is superior to the other, then on average 1 in every 154 matches will end in a draw.

A scoring system must identify a winner in finals matches. In order to make comparisons of scoring systems it is necessary to evaluate various statistical characteristics. We studied the following relevant characteristics a) probability of the superior team winning, b) match duration, c) fairness. Spectator interest is also a consideration. From table 1, the probability of the superior team winning an other final match (0.874) is about the same as that of winning a grand final match (0.875), yet the overall expected match duration is less for other final matches. It could also be argued that replaying matches on the following week (if a draw is obtained) may cause complaints from spectators, given that spectators have paid for an event that has not been completed. It could be argued that the system used in other finals matches is better than the system used in grand final matches since, although it has a slightly smaller probability of correctly identifying the better team, the expected period of time is reduced, and is possibly also better for spectator interest. Table 2 gives the probabilities of other finals matches going for $T_1$ minutes in duration given teams are level after 80 minutes of play. These values can be used as a guide to determine whether the scoring system used in
other finals matches could be applied to round matches to eliminate any possibility of a draw.

New AFL scoring systems were identified in section 7. These systems could be used as possible alternative systems for round matches to eliminate the possibility of a draw, as well as for finals matches as an alternative to the currently used systems. If a draw has occurred after 80 minutes of play in the current system used in other finals matches, the probability of a match going for an extra 20 minutes was evaluated as 0.047 and 0.044 using Parameters (1) and (2) respectively. These probabilities are reduced for the new AFL systems as shown in tables 5 and 6, where the probabilities of a match going for an extra 20 minutes for System L_{16} was evaluated as 0.006 and 0.007 using Parameters (1) and (2) respectively. Such a system could reduce the likelihood of player injuries resulting from ‘long matches’. Note that similar results will occur for System L_{66}.

The probability of the superior team winning other finals matches under the current system is comparable to the probability of the superior team winning under the new AFL systems. The probabilities of playing more time after the fixed 80 minutes of play for the new AFL systems are given in table 9. It was shown that although these new systems are much more likely to go beyond 80 minutes of play (than the current system used in finals matches), the relative size of the additional time required was reduced with these new systems. If a scoring system was required in round matches where a winner had to be determined, then the new AFL systems could possibly be considered to be better than the current system used in finals matches with regard to increasing spectator interest and reducing the likelihood of very long matches occurring. Spectator interest may be increased by having a system in which more time is given for a closely fought match. In System L_{66} for example, about 10% of matches will result in playing more than 80 minutes. Given that the primary objective of AFL is to score goals (rather than behinds), it could seem appropriate to adopt a scoring system where a team must be ahead by at least 6 points (a goal) in order to win the match. It might be argued that the new AFL systems are fairer than the current systems used in AFL, as, for example, in the current systems teams often apply strategies to ‘wind down the clock’ when ahead in the match. Further, decisions on whether points have been scored before or after the siren (which could decide the outcome of the match) can be difficult to determine under the current scoring systems.

9. Conclusions

New AFL scoring systems were devised as possible alternative systems for round matches to stop draws occurring. Firstly, a scoring system was proposed such that if teams are level after the fixed 80 minutes play, then play continues until a team is at least a given number of points ahead and wins the match. A generalization of
this system was then given such that a team had be at least a given number of points ahead after the fixed 80 minutes of play, otherwise play continues until a team is at least a given number of points ahead and wins the match. If a scoring system was required in round matches where a winner had to be determined, then the new AFL systems could possibly be considered to be better than the current system used in finals matches with regard to increasing spectator interest and reducing the likelihood of very long matches occurring. These new systems could also be used as alternative systems in finals matches. The methods used to obtain the winning probabilities could be applied to other sports such as rugby league (3 types of scoring shots) and rugby union (4 types of scoring shots). Further, these sports applications could be used as a teaching exercise in recursion.

**References**


