AN IMPROVEMENT TO THE TENNIS CHALLENGE SYSTEM

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Abstract

Mathematics is applied to the tennis challenge system to derive a fairer challenging method. This method could be used in actual tournament play creating greater spectator interest. The concept of 'importance' is used such that a player has a free challenge if the importance of a point is above a certain threshold.

Keywords: Importance of points, Markov Chain model, player fairness

1. INTRODUCTION

The new challenge system for close line calls in tennis has been used on the ATP and WTA tour for Grand Slam events since the 2006 US Open, and was designed to increase fairness for players by obtaining accurate line calls and enhance spectator interest through video technology. In the current system, players have unlimited opportunity to challenge, but once three incorrect challenges are made in a set, they cannot challenge again until the next set. If the set goes to a tiebreak game, players are given additional opportunities to challenge (usually one extra). If the match is tied at six games all in an advantage set, the counter is reset with both players again having a limit of up to three incorrect challenges in the next 12 games, and this resetting process is repeated after every 12 games.

Strategies as to when players should challenge have recently appeared in the literature. Pollard et al. (2010) show that challenge decisions are based on the rate at which challenges occur, the expected number of points remaining in the set, the number of challenges remaining in the set, the probability of the challenge decision being successful and the importance of the point to winning the set. Clarke and Norman (2010) apply dynamic programming to the challenge system to investigate the optimal challenge strategy and obtain some general rules.

There appears to be problems with the current challenge system:

- Firstly, both of the above articles show that early in the set a player needs to decide whether to challenge or save challenges to later on in the set when the points are typically more important. Having to make such decisions is completely irrelevant to the game of tennis itself. The aim of the contest is to find the better player, and not to favour the player who is luckier within, or better at playing the challenge system. This is reflected by an article *Replay System Becomes Part of Players' Strategies* in The New York Times by Greg Bishop during the 2009 US Open.
  http://www.nytimes.com/2009/09/11/sports/tennis/11challenges.html

- Secondly, a player can run out of challenges because that particular set has a lot of balls that go close to the lines. This is perhaps particularly true in men's singles and men's doubles. The problem is exacerbated when each player does not have a similar number of challenges. A player who plays more balls near the lines is disadvantaged relatively. The player who, by chance has the need for more challenges, is disadvantaged.

- Thirdly, it would appear to be disappointing for the player and the spectators when that player runs out of challenges, the point is very important, and a challenge would have a clear likelihood of success. What is the chance that a grand slam final will be 'messed up' by an umpire making a wrong call and the player having run out of challenges, and subsequently losing the final when he might well have won it otherwise? This would be a very bad result for the player, the umpire and the game. Maybe this probability is not quite as small as some people might expect.

METHODS

a) Markov Chain model

We explain the method by first looking at a single game where we have two players, A and B, and player A has a constant probability p_A of winning a point on serve. We set up a Markov chain model of a game where the state of the game is the current game score in points (thus 40-30 is 3-2). With probability p_A the state changes from a, b to a + 1, b and with probability $q_A=1-p_A$ it changes from a, b to a, b + 1. Thus if $P_A(a,b)$ is the probability that player A wins the game when the score is (a,b), we have:

$P_A(a,b)=p_A P_A(a+1,b)+q_A P_A(a,b+1)$

The boundary values are:
$P_A(a,b) = 1$ if $a = 4, b \le 2$, $P_A(a,b) = 0$ if $b = 4, a \le 2$.

The boundary values and formula can be entered on a simple spreadsheet. The problem of deuce can be handled in two ways. Since deuce is logically equivalent to 30-30, a formula for this can be entered in the deuce cell. This creates a circular cell reference, but the iterative function of Excel can be turned on, and Excel will iterate to a solution. In preference, an explicit formula is obtained by recognizing that the chance of winning from deuce is in the form of a geometric series

$P_A(3,3) = p_A^2 + p_A^2 2p_A q_A + p_A^2 (2p_A q_A)^2 + p_A^2 (2p_A q_A)^3 + \ldots\ldots\ldots$
where the first term is p_A^2 and the common ratio is $2p_A q_A$

The sum is given by $p_A^2/(1-2p_A q_A)$ provided that $-1<2p_A q_A<1$. We know that $0<2p_A q_A<1$, since $p_A>0$, $q_A>0$ and $1-2p_A q_A=p_A^2+q_A^2>0$.

Therefore, the probability of winning from deuce is $p_A^2/(1-2p_A q_A)$. Since $p_A+q_A=1$, this can be expressed as:

$$P_A(3,3) = p_A^2 /(p_A^2+q_A^2)$$

Excel spreadsheet code to obtain the conditional probabilities of player A winning a game on serve is as follows:
Enter $\mathbf{p_A}$ in cell D1
Enter $\mathbf{q_A}$ in cell D2
Enter **0.60** in cell E1
Enter **=1-E1** in cell E2
Enter **1** in cells C11, D11 and E11
Enter **0** in cells G7, G8 and G9
Enter **= E1^2/(E1^2+E2^2)** in cell F10
Enter **=$E$1*C8+$E$2*D7** in cell C7
Copy and Paste cell **C7** in cells D7, E7, F7, C8, D8, E8, F8, C9, D9, E9, F9, C10, D10 and E10

Notice the absolute and relative referencing used in the formula =$E$1*C8+$E$2*D7. By setting up an equation in this recursive format, the remaining conditional probabilities can easily and quickly be obtained by copying and pasting.

Similar recursion formulas with boundary conditions can be obtained for a tiebreak game conditional on the point score, set conditional on the game score and a match conditional on the set score. A predictions model is then applied to estimate the parameters of the probabilities of players winning a point on serve (Barnett et al, 2011).

b) Importance of points

Morris (1977) defines the importance of a point for winning a game (I_{PG}) as the probability that the server wins the game given he wins the next point minus the probability that the server wins the game given he loses the next point. The importance of a point to winning a game is thus:
$I_A(a,b)=P_A(a+1,b)-P_A(a,b+1)$.

Table 1 gives the importance of points to winning the game (I_{PG}) when the server has a 0.62 probability of winning a point on serve, and shows that 30-40 and Ad-Out are the most important points in the game. In a similar way, we can define the importance of a game to winning a set and the importance of a set to winning a match. Table 2 gives the importance of games to winning a tiebreak set (I_{GS}) for player A serving. Player A and Player B were assigned point probabilities of 0.62 and 0.60 respectively to reflect overall averages in men's tennis. It is clear that every point is equally important for both players. Table 2 shows that the tiebreak game has the highest importance of 1.00, as the winner of this game wins the set. Similarly, table 3 gives the importance of sets to winning a best-of-5 set match (I_{SM}) and shows that the deciding set at 2 sets-all has the highest importance of 1.00, as the winner of this set win the match. Morris (1977) derived the following useful multiplicative result to obtain the importance of a point to winning the match (I_{PM}): For any point of any game of any set: $I_{PM} = I_{PG} * I_{GS} * I_{SM}$.

The definition of importance of a point in a match is a way of stating how much difference will result in the outcome of the match depending on whether a point is won or lost. In the context of a challenge system, importance of a point in a match can be viewed by how much percentage error will occur if a wrong decision is made. For example, suppose the score in a best-of-5 set match (all tiebreak sets) is 2-2 in sets, 5-5 in games and 30-30 in points and player A is currently serving. Suppose player A is winning 62% on serve and player B is winning 60% on serve. Using a Markov Chain model (Barnett and Clarke, 2005), player A has a 51.5% chance of winning the match from that position. If player A won the point, then his chance of winning the match would be 60.3%: whereas if player A lost the point then his chance of winning the match would be 37.3%. Therefore, the importance of the point in the match is given as 60.3%-37.3%=23.0%. If a wrong decision was made at that particular point in the match, then it would cost one of the players 23 percentage points in their chance of winning the match.

	Receiver's score					
Server's score		0	15	30	40	Ad
	0	0.25	0.34	0.38	0.28	
	15	0.19	0.31	0.45	0.45	
	30	0.11	0.23	0.45	0.73	
	40	0.04	0.10	0.27	0.45	0.73
	Ad				0.27	

Table 1: Importance of points to winning a game when the server has a 0.62 probability of winning a point on serve

	Player B's score							
Player A's score		0	1	2	3	4	5	6
	0	0.29	0.29	0.22	0.18	0.06	0.02	
	1	0.26	0.32	0.33	0.21	0.16	0.03	
	2	0.25	0.29	0.36	0.37	0.20	0.11	
	3	0.13	0.27	0.33	0.42	0.43	0.14	
	4	0.08	0.11	0.30	0.38	0.52	0.54	
	5	0.01	0.06	0.08	0.34	0.46	0.52	0.53
	6						0.47	1.00

Table 2: Importance of games to winning a tiebreak set when player A and player B have a 0.62 and 0.60 probability of winning a point on service respectively and player A is serving

	B's score			
A's score		0	1	2
	0	0.36	0.42	0.32
	1	0.32	0.49	0.57
	2	0.18	0.43	1.00

Table 3: Importance of sets to winning a best-of-5 set match when player A and player B have a 0.62 and 0.60 probability of winning a point on service respectively

3. RESULTS

a) Proposed new challenge system

It is proposed that the present challenge rule is modified in one way. Namely, that a player is allowed to challenge on points with sufficiently large importance, without risking that player's challenge point total.

Suppose the threshold value on when a player can always challenge a line call was given by the importance of the point in the match at 2 sets-all, 3 games-all, 0 points-all and player A serving. This is calculated as 1.00*0.42*0.25=0.104 when player A and player B have a 0.62 and 0.60 probability of winning a point on serve respectively. Then a player can always challenge at 2 sets-all and 3 games-all, only if the point score in the match has an importance of at least 0.104. This occurs at 2 sets-all and 3 games-all for 30-40 or Ad-Out (I_{PM}=0.305), 15-40 (I_{PM}=0.189), 15-30 (I_{PM}=0.188), 30-30 or deuce (I_{PM}=0.187), 0-30 (I_{PM}=0.161), 0-15 (I_{PM}=0.143), 15-15 (I_{PM}=0.132), 0-40 (I_{PM}=0.117), 40-30 or Ad-In (I_{PM}=0.115) and 0-0 (I_{PM}=0.104). This is represented in table 4 for a range of game scores in the deciding set, where an X indicates that a challenge is always allowable by both players. Note that a player can challenge at 2 sets-all and 6 games-all (tiebreak game), only if the point score has an importance of at least 0.231. This occurs for the majority of points in the tiebreak game, as expected.

	Score line at 2 sets-all (player A serving)					
Point score	0-0	1-1	2-2	3-3	4-4	5-5
30-40 or Ad-Out	X	X	X	X	X	X
15-40	X	X	X	X	X	X
15-30	X	X	X	X	X	X
30-30 or Deuce	X	X	X	X	X	X
0-30	X	X	X	X	X	X
0-15		X	X	X	X	X
15-15			X	X	X	X
0-40				X	X	X
40-30 or Ad-In				X	X	X
0-0				X	X	X
30-15					X	X
15-0, 30-0, 40-15 or 40-0						

Table 4: Indication as to whether a player can always challenge on a particular point in a match for a range of game scores in the deciding set given that the threshold value is given as 0.104

4. DISCUSSION

Being able to challenge 'free of charge' on some point scores later in the set, but not earlier, might cause confusion for some players in some situations. To get around this problem, you introduce a "challenge" screen visible to both players which gives a green light before the point is played if the point has a sufficient level of 'importance'. Otherwise the screen is empty (or a red light). Spectator's interest would also be lifted, quite possibly or naturally. It would give commentators an additional thing to talk about being the importance of points. Note that the free challenge light going on could be automated with the umpire's score card.

Instead of giving three incorrect challenges per set as proposed above, suppose players are given x challenges per set and have unlimited opportunity to challenge, but once x incorrect challenges are made in a set, they cannot challenge again until the next set. Further, players can always challenge when the point has a sufficient level of 'importance' = y without affecting their challenge point total, otherwise players cannot challenge if they have run out of their challenge point total.

Scenario 1)
When x=3 and the level of 'importance'=1, is equivalent to the current system.

Scenario 2)
When x=0 and the level of 'importance'=y, is "optimally" the best system in terms of minimizing time on player's challenging on "unimportant" points.

Scenario 3)
When $1 \leq x \leq 3$ and the level of 'importance'=y, is somewhere between Scenario 1) and Scenario 2)

At the very least Scenario 3) could be adopted such that players can always challenge when the point has a sufficient level of 'importance' = y without affecting their challenge point total. However, Scenario 2) could be obtained as an "optimal" system in terms of minimizing time on player's challenging on "unimportant" points.

However, whilst the fifth set is the most important set, it may be better to have the same procedure in each set. This is likely to be more easily accepted by the relevant people. An advantage of this is that the operation of the system would be identical for all sets. There is something nice about uniformity. Further, under the system described above, there would be points in earlier sets that are more important than some of the 'free challenge' points in the fifth set. This may present a problem. So just looking at set importances rather than match importances could be a preference.

Maybe every point in the tiebreak game should be a free challenge (with no 'additional' challenges given to the players at 6/6 because the set is 'long') and any point within the set at least as important as any point in the tiebreak game should also be free. This could be a useful selling point to the interested parties. If this was considered too generous, any point at least as important as say 2/2 within the tiebreak game could be a free challenge. The fact that players are given an extra challenge at 6/6 gives some merit to the ideas in this paper. The idea in this paper in fact parallels the present rules at 6/6. It just formalizes some present operational characteristics.

If the "challenge" screen was too much of a problem for players then you could have a system where a free challenge was given on every point in tiebreak games (representing a high level of importance) and at say set/match points.

5. CONCLUSIONS

Throughout this article it is demonstrated that a fairer method to the line call challenge system is such that a player should always be allowed to challenge at a score line with a certain level of importance without affecting their challenge point total.

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